Numerical Analysis of the Vibration of an Elastic Beam using a Wake Oscillator Model

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Elastic structures submerged in flowing fluids, such as oil risers and turbine blades, experience Vortex-Induced Vibration (VIV). In its turn, vibration causes fatigue damage reducing the working life of structures. This phenomenon is known as fluid-structure problem.
Introduction

- The fluid-structure problem requires the numerical solution of the Navier-Stokes equations coupled with the equations of elasticity for the structure. However, a lot of computational power is required. Then, simplified models not be very computationally demanding which simulate the fluid-structure interaction of elastic structures are required.
Introduction

- Sometimes the fluid-structure problems is addressed by introducing and additional van der Pol equation in the structure elasticity equation. These models are known as wake-oscillator models. Both the beam equation and the wake-oscillator model are coupled through the acceleration term.
Introduction

- Traditionally, wake-oscillator models are applied to long slender structures such as marine cables, oil risers, and so on. In this work the vibration behavior and the energy exchange among modes of a non-slender elastic beam immersed in a moving fluid and suffering VIV is analyzed using a wake oscillator model.
Mathematical Model

- The equation which governs the dynamics of a straight cylinder oscillating in the direction transverse to the flow is required.
- To describe the dynamics of the cylinder’s wake, a forced van der Pol oscillator equation is employed to determine the local lift coefficient.
Mathematical Model

- The transversal displacement of the elastic beam suffering VIV is coupled to the forces of the flowing fluid by the acceleration term of the local lift coefficient.
Numerical Solution

- Finite differences is a proved method to solve vibration problems, and is employed here. To assure stability and convergence of the numerical results, computer runs were made employing mesh sizes of 0.01 and time step of $2 \times 10^{-6}$. Runs were made for a dimensionless time long enough to guarantee that steady state is reached.
Results

- Uniform flow is considered in this work. Three vibration modes corresponding to the non-excited elastic beam were considered in the analysis, whose eigenfrequencies are 0.56, 3.51 and 9.82 Hz, respectively.
Results

- The Strouhal number relates the frequency of shedding to the velocity of the flow and a characteristic dimension of the body. The Strouhal number for a cylinder is 0.2 over a wide range of flow velocities.
Results

- A tracking point was located at the tip of the beam in order to study the energy transfer among modes and determine the power spectrum from the tip time series.
Results

- It is analyzed the effect of:
  - Initial excited mode
  - Initial velocity
  - Shedding pulation
  - Mass ratio
Results

- Time series of the tip position when the second (left) and third (right) modes are excited. Zero initial velocity.
Results

- Time series of the beam tip transverse position for an initial velocity of 5 when the third mode is excited (left). Corresponding power spectrum (right).
Results

- Main frequency as function of the initial velocity.
Results

- Effect of the shedding pulsation on the tip oscillation amplitude. Excited mode: first (circle), second (square), third (triangle).
Results

- Vibration amplitude of the beam tip as function of the mass ratio. Excited mode: second (square), third (triangle).
Conclusions

- The vibration of an elastic beam suffering vortex-induced vibration is analyzed here using a wake-oscillator model. Numerical solutions show transfer of energy among modes and multiple frequencies vibration. In accordance with the computer results, the excited mode and the initial velocity influence the dominant frequency whereas the shedding pulsation influences the vibration amplitude.
Thanks

- Questions?